*Definition:*  *distribution*

In a sequence of independent Bernoulli trials with success probability ,

if is the number of failures before the th success,

then has the negative binomial distribution with parameters and ,

i.e.

Let .

If (counting failures before the first success), then

,

.

We can consider as the sum of iid RVs, so

,

,

.

*Definition: distribution, i.e. mean parameterization of NB distribution*

In the original distribution,

, .

Reparameterize by letting , so that and, and we have

.

We say that .

In this case, .

*Extension of NB2 distribution to*

Using the gamma function, we can allow for positive real values of :

, .

It may require proving that the expectation and variance are still the same for noninteger .

*Theorem: limit of is*

As , the distribution approaches the distribution.

Equivalently, the distribution approaches the distribution.

*Proof*

For ,

by algebraic rearrangement.

Since

.

we have .

Taking limits as ,

,

which is the PMF. ■

*Definition: zero-inflated distribution*

In a zero-inflated negative binomial model, we hypothesize that is generated:

\* by a zero process with probability , and

\* by a negative binomial process with probability .

If is the PMF, then the PMF is given by

.